

from Eq. (8) in terms of γ_w or γ , is at least approximately independent of volume and temperature.

One notes that Eq. (51) predicts a positive value of $d\gamma_0/d\ln v$. On the basis of the Lindemann law, the author has shown that the curvature of the fusion curve can be normal in the sense of Bridgman only if $\gamma - \frac{1}{3} + d\gamma/d\ln V$ is positive for the solid at fusion³⁰; the sign of the derivative implied by Eq. (51) thus is consistent with this result. In a comparison with experiment of a form of Simon's semiempirical fusion equation obtained theoretically, the author has determined values of $d\ln(\gamma - \frac{1}{3})/d\ln V$ for the alkali metals.²⁰ The values are positive, consistently with Eq. (51), and are of the order implied by Eq. (56) for n and m chosen to correspond to the Birch equation.

It can be noted that the generalized equation of state obtained is concordant with, but is not restricted by, the validity of Grüneisen's law. This conclusion follows from the fact that the basic relation (17), to which the generalized equation of state conforms, is a thermodynamic identity independent of a model. For a metal at sufficiently low temperature, Grüneisen's law fails in the sense that the Grüneisen parameter requires a correction for the contribution of the electrons to the thermal pressure³⁵; for a similar reason, the law is generally not valid for a superconductor. In neither case should the generalization to arbitrary temperature involved in Eq. (15) fail to be applicable within the approximations made, if the form (4) is valid under isothermal conditions.

V. COMPARISON WITH EXPERIMENTAL DATA

Results of Swenson⁹ for the compression of the alkali metals at two temperatures (77 and 4.2°K) can be used to compare predictions of the generalized Birch equation with experiment. The element potassium will be chosen for the check, since the corresponding values of the parameter ξ of the correction factor (5) vanish at both temperatures.

Swenson expresses his results for the pressure as a function of compression by tabulating values of the density and the compressibility K_0^{-1} at zero pressure for the two temperatures in question, which fix the constants V_0 (for unit mass) and K_0 of the isothermal Birch equation as fitted to his data. The values of the latter pair of constants corresponding to the former are shown in the first two columns of Table I, for

TABLE I. Parameters of the generalized Birch equation of state for potassium, from data of Swenson.

T °K	V_0 cm ³ /g	K_0 10 ⁴ atmos	α_0 (°C) ⁻¹	$\eta_0 \alpha_0$ (°C) ⁻¹	η_0
77	1.09 ₂	3.38	0.0001 ₆	0.0003 ₇	2.4
4.2	1.08 ₁	3.47			

³⁵ S. Visvanathan, Phys. Rev. 81, 626 (1951); J. J. Gilvarry, Phys. Rev. 102, 317 (1956).

potassium at the two temperatures. On Swenson's definitions, V_0 and K_0 depend on the temperature; hence V_0 and K_0 represent values of \mathcal{V} and \mathcal{K} , respectively, in the notation of this paper. The pressure for potassium is shown in Fig. 1 as a function of the relative compression $(\mathcal{V}-V)/\mathcal{V}$ for the two temperatures, as computed from the isothermal Birch equation with constants from Table I.

The values of the parameters α_0 , $\eta_0 \alpha_0$, and η_0 for potassium are shown in the last three columns of Table I, as implied by the constants in the first two columns. Since these determinations correspond directly to the data of Swenson, it is clear that values at the two temperatures of \mathcal{V} and \mathcal{K} from Eqs. (27) and (32), respectively, merely reproduce the values of V_0 and K_0 in the first two columns of Table I. Thus, a substantive check of the generalized Birch equation cannot

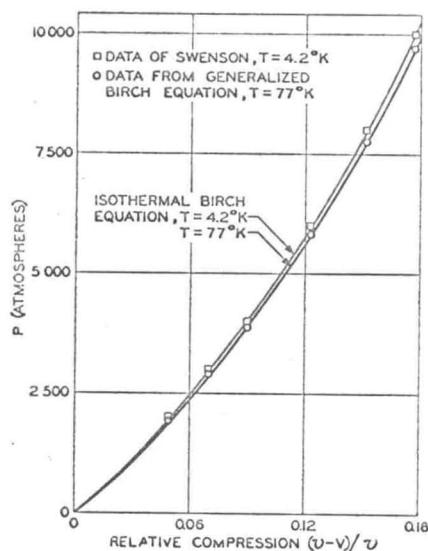


FIG. 1. Pressure as a function of the relative compression for potassium, as computed from the isothermal Birch equation with data from Table I, for comparison with data points at 77°K predicted by the generalized Birch equation from values at 4.2°K.

be obtained directly, since its form with parameters evaluated numerically for one of the two temperatures is simply the isothermal form with constants equal to those of Swenson.

However, Swenson tabulates smoothed experimental data for potassium at one temperature, 4.2°K. An independent check of the generalized Birch equation can be obtained by using it to predict pressures at 77°K from the tabular values for 4.2°K, given as a function of $(V_0 - V)/V_0 = (\mathcal{V} - V)/\mathcal{V}$. For a fixed value of the last quantity, Eq. (15) states that $P(T)$ is given in terms of $P(T_0)$ by

$$P(T) = [1 - \eta_0 \alpha_0 (T - T_0)] P(T_0). \quad (38)$$

Use of this equation with values of parameters from Table I to predict pressures at 77°K from those at 4.2°K yields excellent agreement with results from the

isothermal Birch equation shown in Fig. 1. The degree of agreement is clearly shown in Fig. 2, where the difference in pressure $P(4.2^\circ\text{K}) - P(77^\circ\text{K})$ is plotted against relative compression. The data points from Swenson (open circles) are compared with the values predicted by the generalized Birch equation (open squares). The agreement is excellent, especially at lower pressures.

The degree of agreement is clearly shown in Fig. 2, where the difference in pressure $P(4.2^\circ\text{K}) - P(77^\circ\text{K})$ is plotted against relative compression. The data points from Swenson (open circles) are compared with the values predicted by the generalized Birch equation (open squares). The agreement is excellent, especially at lower pressures.

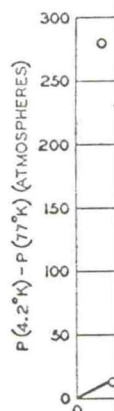


FIG. 2. Difference in pressure $P(4.2^\circ\text{K}) - P(77^\circ\text{K})$ versus relative compression for potassium, as computed from the isothermal Birch equation with data from Table I, for comparison with data points at 77°K predicted by the generalized Birch equation from values at 4.2°K.

comparison with Swenson's data. The agreement with the Birch equation is excellent.

It is clear that the results can be used directly to predict corresponding pressures at 77°K from those at 4.2°K, given as a function of $(V_0 - V)/V_0 = (\mathcal{V} - V)/\mathcal{V}$. For a fixed value of the last quantity, Eq. (15) states that $P(T)$ is given in terms of $P(T_0)$ by